MATH2050C Assignment 7

Deadline: March 7, 2018.

Hand in: 3.7 no. 3c, 10, 15, 16.

Section 3.7 no. 3ac, 7, 10, 11, 12, 15, 16.

Supplementary Exercises

1. Show that for any sequence, there associates an infinite series whose convergence/divergence is the same as that for the sequence.

2. An infinite series $\sum_n a_n$ is called **absolutely convergent** if $\sum_n |a_n|$ is convergent. Show that an absolutely convergent infinite series is convergent but the convergence is not always true.

Essential 3.7 Infinite Series

In this section you should know

- Definition of the convergence of an infinite series.
- The convergence/divergence of $\sum_n 1/n^p$.

Let $\sum_{n=1}^{\infty} a_n$ be an infinite series. It n-th partial sum s_n is given by $\sum_{k=1}^{n} a_k$. An infinite series $\sum_{n=1}^{\infty} a_n$ is called **convergent/divergent** if the sequence $\{s_n\}$ is convergent/divergent. When an infinite series converges, we use $\sum_{n=1}^{\infty} a_n$ to denote the limit $\lim_{n\to\infty} s_n$. Thus, the notation $\sum_{n=1}^{\infty} a_n$ has two meanings; first it is the notation for an infinite series, and second, it is the ultimate sum of the infinite series (provided it converges).

Proposition 7.1. An infinite series is convergent if and only if, for every $\varepsilon > 0$, there is some n_ε such that

$$|\sum_{k=m}^{n} a_k| < \varepsilon \ , \quad \forall n, m \geq n_\varepsilon \ .$$

Proof. Apply Cauchy Convergence Criterion to $\{s_n\}$.

An application of this proposition is to show that the harmonic series $\sum_n 1/n$ is divergent. Can you reproduce the proof here?

Proposition 7.2. For any convergent infinite series $\sum_n a_n$, the sequence $\{a_n\}$ converges to 0.

Proof. Apply Proposition 7.1 by taking $n = m + 1$.

Proposition 7.3. The infinite series $\sum_n 1/n^p$ is convergent if and only if $p > 1$.

Refer to text for the proof. This result follows from the Integral Test easily (but wait until MATH2060).